2.4.1 Exercises

To see all of the help resources associated with this section, click OSttS Chapter 2.

In Exercises 1 - 32, solve the inequality algebraically and check graphically. Write your answer using interval notation.

For help with these exercises, click on one or more of the resources below:

- Solving inequalities involving the absolute value
- Solving inequalities involving quadratic functions using sign diagrams
- Understanding inequalities graphically

1. $|3x - 5| \leq 4$
2. $|7x + 2| > 10$
3. $|2x + 1| - 5 < 0$
4. $|2 - x| - 4 \geq -3$
5. $|3x + 5| + 2 < 1$
6. $2|7 - x| + 4 > 1$
7. $2 \leq |4 - x| < 7$
8. $1 < |2x - 9| \leq 3$
9. $|x + 3| \geq |6x + 9|$
10. $|x - 3| - |2x + 1| < 0$
11. $|1 - 2x| \geq x + 5$
12. $x + 5 < |x + 5|$
13. $x \geq |x + 1|$
14. $|2x + 1| \leq 6 - x$
15. $x + |2x - 3| < 2$
16. $|3 - x| \geq x - 5$
17. $x^2 + 2x - 3 \geq 0$
18. $16x^2 + 8x + 1 > 0$
19. $x^2 + 9 < 6x$
20. $9x^2 + 16 \geq 24x$
21. $x^2 + 4 \leq 4x$
22. $x^2 + 1 < 0$
23. $3x^2 \leq 11x + 4$
24. $x > x^2$
25. $2x^2 - 4x - 1 > 0$
26. $5x + 4 \leq 3x^2$
27. $2 \leq |x^2 - 9| < 9$
28. $x^2 \leq |4x - 3|$
29. $x^2 + x + 1 \geq 0$
30. $x^2 \geq |x|$
31. $x|x + 5| \geq -6$
32. $x|x - 3| < 2$

33. The profit, in dollars, made by selling x bottles of 100% All-Natural Certified Free-Trade Organic Sasquatch Tonic is given by $P(x) = -x^2 + 25x - 100$, for $0 \leq x \leq 35$. How many bottles of tonic must be sold to make at least \$50 in profit?

34. Suppose $C(x) = x^2 - 10x + 27$, $x \geq 0$ represents the costs, in *hundreds* of dollars, to produce x *thousand* pens. Find the number of pens which can be produced for no more than \$1100.

35. The temperature T, in degrees Fahrenheit, t hours after 6 AM is given by $T(t) = -\frac{1}{2}t^2+8t+32$, for $0 \leq t \leq 12$. When is it warmer than 42° Fahrenheit?

36. The height h in feet of a model rocket above the ground t seconds after lift-off is given by $h(t) = -5t^2 + 100t$, for $0 \leq t \leq 20$. When is the rocket at least 250 feet off the ground? Round your answer to two decimal places.

37. If a slingshot is used to shoot a marble straight up into the air from 2 meters above the ground with an initial velocity of 30 meters per second, for what values of time t will the marble be over 35 meters above the ground? (Refer to Exercise 25 in Section 2.3 for assistance if needed.) Round your answers to two decimal places.

38. What temperature values in degrees Celsius are equivalent to the temperature range $50°F$ to $95°F$? (Refer to Exercise 35 in Section 2.1 for assistance if needed.)

In Exercises 39 - 42, write and solve an inequality involving absolute values for the given statement.

39. Find all real numbers x so that x is within 4 units of 2.

40. Find all real numbers x so that $3x$ is within 2 units of -1.

41. Find all real numbers x so that x^2 is within 1 unit of 3.

42. Find all real numbers x so that x^2 is at least 7 units away from 4.

43. The surface area S of a cube with edge length x is given by $S(x) = 6x^2$ for $x > 0$. Suppose the cubes your company manufactures are supposed to have a surface area of exactly 42 square centimeters, but the machines you own are old and cannot always make a cube with the precise surface area desired. Write an inequality using absolute value that says the surface area of a given cube is no more than 3 square centimeters away (high or low) from the target of 42 square centimeters. Solve the inequality and write your answer using interval notation.

44. Suppose f is a function, L is a real number and ε is a positive number. Discuss with your classmates what the inequality $|f(x) - L| < \varepsilon$ means algebraically and graphically.[6]

[6] Understanding this type of inequality is really important in Calculus.

In Exercises 45 - 50, sketch the graph of the relation.
For help with these exercises, click on one or more of the resources below:

- Graphing inequalities in two variables in the plane
- Using inequalities to describe regions in the plane

45. $R = \{(x, y) : y \leq x - 1\}$
46. $R = \{(x, y) : y > x^2 + 1\}$
47. $R = \{(x, y) : -1 < y \leq 2x + 1\}$
48. $R = \{(x, y) : x^2 \leq y < x + 2\}$
49. $R = \{(x, y) : |x| - 4 < y < 2 - x\}$
50. $R = \{(x, y) : x^2 < y \leq |4x - 3|\}$
51. Prove the second, third and fourth parts of Theorem 2.4.

Checkpoint Quiz 2.4

1. Solve: $\frac{1 - x}{2} \leq |x + 2|$. Check your answer graphically.
2. Solve: $16x^2 \leq 24x - 9$
3. Solve: $x^2 > 2(x + 1)$
4. Solve: $x^2 - 6 \leq x + |x - 3|$.
5. Sketch the relation: $R = \{(x, y) : x^2 \leq y < x + 6\}$

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2
- Quiz Solution Part 3
- Quiz Solution Part 4

2.4.2 ANSWERS

1. $\left[\frac{1}{3}, 3\right]$
2. $\left(-\infty, -\frac{12}{7}\right) \cup \left(\frac{8}{7}, \infty\right)$
3. $(-3, 2)$
4. $(-\infty, 1] \cup [3, \infty)$
5. No solution
6. $(-\infty, \infty)$
7. $(-3, 2] \cup [6, 11)$
8. $[3, 4) \cup (5, 6]$
9. $\left[-\frac{12}{7}, -\frac{6}{5}\right]$
10. $(-\infty, -4) \cup \left(\frac{2}{3}, \infty\right)$
11. $\left(-\infty, -\frac{4}{3}\right] \cup [6, \infty)$
12. $(-\infty, -5)$
13. No Solution.
14. $\left[-7, \frac{5}{3}\right]$
15. $\left(1, \frac{5}{3}\right)$
16. $(-\infty, \infty)$
17. $(-\infty, -3] \cup [1, \infty)$
18. $\left(-\infty, -\frac{1}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$
19. No solution
20. $(-\infty, \infty)$
21. $\{2\}$
22. No solution
23. $\left[-\frac{1}{3}, 4\right]$
24. $(0, 1)$
25. $\left(-\infty, 1 - \frac{\sqrt{6}}{2}\right) \cup \left(1 + \frac{\sqrt{6}}{2}, \infty\right)$
26. $\left(-\infty, \frac{5-\sqrt{73}}{6}\right] \cup \left[\frac{5+\sqrt{73}}{6}, \infty\right)$
27. $\left(-3\sqrt{2}, -\sqrt{11}\right] \cup \left[-\sqrt{7}, 0\right) \cup \left(0, \sqrt{7}\right] \cup \left[\sqrt{11}, 3\sqrt{2}\right)$
28. $\left[-2 - \sqrt{7}, -2 + \sqrt{7}\right] \cup [1, 3]$
29. $(-\infty, \infty)$
30. $(-\infty, -1] \cup \{0\} \cup [1, \infty)$
31. $[-6, -3] \cup [-2, \infty)$
32. $(-\infty, 1) \cup \left(2, \frac{3+\sqrt{17}}{2}\right)$
33. $P(x) \geq 50$ on $[10, 15]$. This means anywhere between 10 and 15 bottles of tonic need to be sold to earn at least \$50 in profit.
34. $C(x) \leq 11$ on $[2, 8]$. This means anywhere between 2000 and 8000 pens can be produced and the cost will not exceed \$1100.
35. $T(t) > 42$ on $(8 - 2\sqrt{11}, 8 + 2\sqrt{11}) \approx (1.37, 14.63)$, which corresponds to between 7:22 AM (1.37 hours after 6 AM) to 8:38 PM (14.63 hours after 6 AM.) However, since the model is valid only for t, $0 \leq t \leq 12$, we restrict our answer and find it is warmer than 42° Fahrenheit from 7:22 AM to 6 PM.

36. $h(t) \geq 250$ on $[10 - 5\sqrt{2}, 10 + 5\sqrt{2}] \approx [2.93, 17.07]$. This means the rocket is at least 250 feet off the ground between 2.93 and 17.07 seconds after lift off.

37. $s(t) = -4.9t^2 + 30t + 2$. $s(t) > 35$ on (approximately) $(1.44, 4.68)$. This means between 1.44 and 4.68 seconds after it is launched into the air, the marble is more than 35 feet off the ground.

38. From our previous work $C(F) = \frac{5}{9}(F - 32)$ so $50 \leq F \leq 95$ becomes $10 \leq C \leq 35$.

39. $|x - 2| \leq 4$, $[-2, 6]$

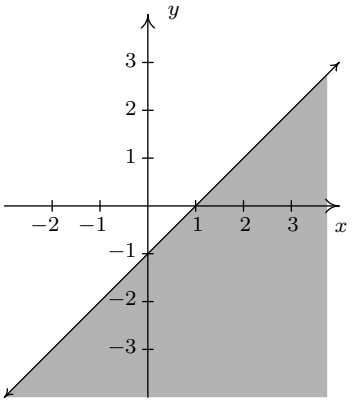
40. $|3x + 1| \leq 2$, $\left[-1, \frac{1}{3}\right]$

41. $|x^2 - 3| \leq 1$, $[-2, -\sqrt{2}\,] \cup [\sqrt{2}, 2]$

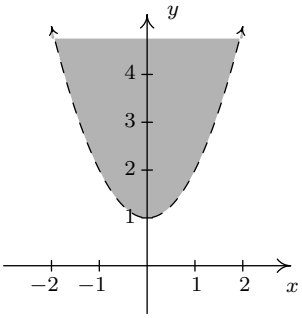
42. $|x^2 - 4| \geq 7$, $(-\infty, -\sqrt{11}\,] \cup [\sqrt{11}, \infty)$

43. Solving $|S(x) - 42| \leq 3$, and disregarding the negative solutions yields $\left[\sqrt{\frac{13}{2}}, \sqrt{\frac{15}{2}}\right] \approx [2.550, 2.739]$. The edge length must be within 2.550 and 2.739 centimeters.

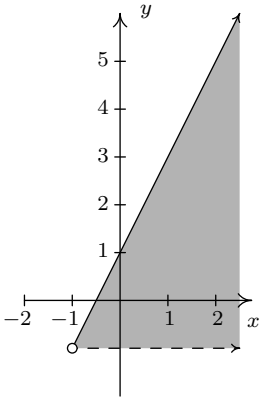
45.



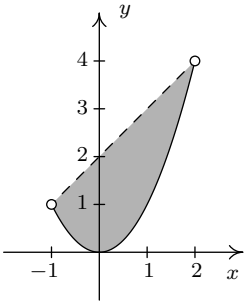
46.



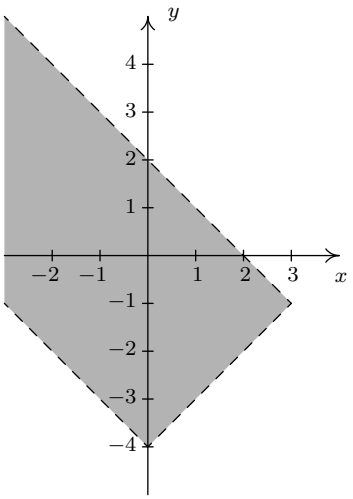
47.



48.



49.



50.